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EFFICIENCY RESEARCH OF THE THREE-LEVEL MODEL OF SMALL-SERIES PRODUCTION PLANNING

We consider the problem of finding an order portfolio that maximizes the total profit according to one of five optimization criteria and should fit the beginning date of the planned period and the due dates specified by the customers. Also, we need to build for this order portfolio a feasible (not violating the due dates) operational plan of jobs processing that would correspond to the minimum possible processing time of the entire order portfolio. We show that the problem in this formulation is a multi-stage scheduling problem. We describe previously developed methodology for the problem solving: the three-level model of production planning. We substantiate the possibility of applying the methodology for any type of small-series production according to one of the five criteria of optimality. We show that independently of the production type considered, whatever is the original production technology, and no matter how the multi-stage scheduling problem is implemented, we reduce the planning problem solving for any of the five optimality criteria to obtaining a feasible solution of the multi-stage scheduling problem for the criterion of maximizing the start time of the earliest job. We show that the efficiency of the multi-stage scheduling problem solving depends on the efficiency of solving the first level of the three-level model. Therefore, we statistically investigate and prove the efficiency of solving the problem of minimizing the total weighted completion time of jobs with precedence relations on a single machine. We show the efficiency of PSC-algorithm for the problem solving for the case when the weights of only terminal vertices of the precedence graph are non-zero. We have shown that the approximation algorithm for this problem solving allows to solve real practical large size problems (we checked dimensions of up to 10,000 jobs). The solutions obtained by the approximation algorithm coincided with those obtained by the exact PSC-algorithm in 99.97 % cases.

Keywords: production planning, PSC-algorithm, exact algorithm, approximation algorithm, combinatorial optimization, scheduling

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ДОСЛІДЖЕННЯ ЕФЕКТИВНОСТІ ТРИРІВНЕВОЇ МОДЕЛІ ПЛАНУВАННЯ ДРІБНОСЕРІЙНОГО ВИРОБНИЦТВА

Розглядається задача знаходження портфеля замовлень, який максимізує сумарний прибуток за одним з п'яти критеріїв оптимізації та при якому дотримано початок планового періоду і директивні строки, задані замовниками. Також потрібно побудувати для цього портфеля замовлень допустимий (що не порушує директивних строків) пооопераційний план виконання робіт, якому відповідав би мінімально можливий час виконання всього портфеля замовлень. Показано, що задача в такій постановці є багатоетапною задачею календарного планування. Описується раніше розроблена методологія розв'язання задачі – трирівнева модель планування виробництва. Обґрунтовується можливість застосування методології для будь-якого виду дрібносерійного виробництва за одним з даних п'яти критеріїв оптимальності. Показано, що для будь-якого виду виробництва, при будь-якій вихідній технології виконання виробів і при будь-якій реалізації багатоетапної задачі календарного планування, розв'язання задачі планування за одним з цих п'яти критеріїв оптимальності зводиться до отримання припустимого розв'язку багатоетапної задачі календарного планування за критерієм максимізації моменту запуску найбільш ранньої роботи. Показано, що ефективність розв'язання багатоетапної задачі календарного планування залежить від ефективності розв'язання першого рівня трирівневої моделі. Тому, статистично досліджується і обґрунтовується ефективність розв'язання задачі мінімізації сумарного зваженого моменту закінчення виконання робіт з відносинами передування на одному приладі. Показана ефективність ПДС-алгоритму розв'язання задачі для випадку, коли ваги всіх вершин графа передування, крім кінцевих, дорівнюють нулю. Показано, що наближений алгоритм розв'язання цієї задачі дозволяє розв'язувати реальні практичні задачі великої розмірності (перевірялися розмірності до 10,000 робіт). Розв'язки, отримані наближеним алгоритмом, збіглися з отриманими точним ПДС-алгоритмом в 99.97 % випадків.

Ключові слова: планування виробництва, ПДС-алгоритм, точний алгоритм, наближений алгоритм, комбінаторна оптимізація, складання розкладів

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ИССЛЕДОВАНИЕ ЭФФЕКТИВНОСТИ ТРЕХУРОВНЕВОЙ МОДЕЛИ ПЛАНИРОВАНИЯ МЕЛКОСЕРИЙНОГО ПРОИЗВОДСТВА

Рассматривается задача нахождения портфеля заказов, максимизирующего суммарную прибыль по одному из пяти критериев оптимизации, при котором соблюдено начало планового периода и директивные сроки, заданные заказчиками. Также требуется построить для этого портфеля заказов допустимый (не нарушающий директивных сроков) пооперационный план выполнения работ, которому соответствует минимально возможное время выполнения всего портфеля заказов. Показано, что задача в такой постановке является многоэтапной задачей календарного планирования. Описывается ранее разработанная методология решения задачи – трехуровневая модель планирования производства. Обосновывается возможность применения методологии для любого вида мелкосерийного производства по любому из данных пяти критериев оптимальности. Показано, что для любого вида производства, при любой исходной технологии выполнения изделий и при любой реализации многоэтапной задачи календарного планирования, решение задачи планирования по любому из пяти этих критериев оптимальности сводится к получению допустимого решения многоэтапной задачи календарного планирования по критерию максимизации момента запуска самой ранней работы. Показано, что эффективность решения многоэтапной задачи календарного планирования зависит от эффективности решения первого уровня трехуровневой модели. Поэтому, статистически исследуется и обосновывается эффективность решения задачи минимизации суммарного взвешенного момента окончания выполнения работ с отношениями предшествования на одном приборе. Показана эффективность ПДС-алгоритма решения задачи для случая, когда веса всех вершин графа предшествования, кроме конечных, равны нулю. Показано, что приближенный алгоритм решения этой задачи позволяет решать реальные практические задачи большой размерности (проверялись размерности до 10,000 работ). Решения, полученные приближенным алгоритмом, совпали с полученными точным ПДС-алгоритмом в 99.97 % случаев.

Ключевые слова: планирование производства, ПДС-алгоритм, точный алгоритм, приближенный алгоритм, комбинаторная оптимизация, составление расписаний

Introduction. Production planning in the current conditions of tough market competition is a complex task that requires consideration of a real technological process' complexity, on the one hand, and the implementation of sophisticated optimization algorithms, on the other hand. Due to the task's complexity, problems of production plan optimization occupy the minds of scientists for about 70 years. Economic and production criteria which became important relate to [1]: profit maximization, costs minimization, orders fulfillment just in time, energy resources saving through the efficient equipment usage, the maximum shortening in the production cycle of products.

According to the principle of hierarchical planning [2–6] which represents a philosophy to address complex problems for a wide variety of systems, authors of [7–9] have proposed a three-level planning model for small-series productions. It was aimed to solve the problem of building a coordinated schedule of jobs processing by a set of enterprise resources. The functional diagram of the model is presented in [8]. Also, they develop a system of interrelated models and methods which allow to take into account the complexity of a modern production and to obtain close to optimal solutions to the planning problem, due to the global optimum search strategy. The first level of the model is based on solving the problem of the total weighted completion time of tasks minimization (TWCT) for the case when only terminal vertices of the precedence relations graph have a non-zero weight coefficient (TWCTZ problem). The TWCTZ problem solution determines the tasks priorities and minimizes the time they pass through the production cycle.

The proposed models and methods are universal in nature and can be implemented for planning in organizational or production systems in various sectors of a national economy, in particular [1, 7], for planning of discrete type production, building industry, project management. The developed models and methods of hierarchical planning can also be applied in computer-aided design systems, information control systems, scientific research automation systems, etc.

The purpose of this article is to substantiate the efficiency of using the methodology developed in [7] to obtain an operational plan for arbitrary objects with a network representation of technological processes. We show that the efficiency of the problem solving is determined by the efficiency of solving TWCTZ problem. Then, we carry out the efficiency study of its solving algorithms.

The Problem Statement. Suppose that we have an order portfolio [7–9] which is a set of n packages of interrelated jobs $J = (J_1, J_2, \dots, J_n)$. We call a package J_i, $i = \overline{1,n}$, a product (under a product we may also mean the entire series of uniform products). Customer specifies for each product the technology of its production and the desired optimization criterion (one of the five basic criteria listed below), as well as the beginning date for the planning period. Also, the customer sets the due dates d_i for all products in accordance with their optimization criteria, except for the case of optimization according to the first basic criterion. On each subset J_i, a partial order is given by an oriented acyclic graph. The partial ordering is obviously determined by the technology of processing the job packages. Each job can begin only after completion of its predecessors. The vertices of the graph correspond to the jobs, the edges indicate the precedence relations. The terminal vertices correspond to the completion of the products processing. For each vertex j of the graph, we know the deterministic processing time l_j (an integrated value indicating the allocated resources: material, human, production, etc.; the critical path of each product determines its processing time). Also, we are given a weight ω_i for each job $i \in I$ where I is the set of terminal vertices identified with a set of products. The value of weight is determined by the potential complexity and importance of those jobs, without which, in general, the product cannot be released. Also, an ambiguity of the jobs related to the need of obtaining a new scientific solution may affect the weight's value. Jobs are processed by a limited set of resources divided into separate, sufficiently autonomous units: *multi-resources*. A multi-resource is a stable group of shared resources, for example, a brigade, a group of equipment of the same type, single-profile subdivision. Multi-resources can be physically located in the same or in different organizations. In the general case, a multi-resource may include equipment of different types. This is determined by the production need or if it allows more efficient fulfillment of the specified orders.

We need:

- to find an order portfolio that maximizes the total profit according to the chosen optimization criterion; it should fit the beginning date of the planned period and the due dates specified by the customers;
- to build for this order portfolio a feasible (not violating the due dates) operational plan of the jobs packages processing on multi-resources; it should maximize the start time of the earliest job, i.e. the time corresponding to the minimum possible processing time of the entire order portfolio.

The five basic optimization criteria are the enterprise's total profit maximization for the following five cases:

1) in the absence of product's due dates. It is shown in [7, 10] that the criterion in this case is equivalent to the total weighted completion time of products minimization criterion with a partial order given on the set of jobs of each product (the TWCT problem);

2) subject to the condition: each product $i \in I$ has a due date d_i that must not be violated (just in time planning).

3) subject to the conditions: each product $i \in I$ has a due date d_i; the total weighted tardiness of tasks in regard to their due dates must be minimized.

4) subject to the conditions: each product $i \in I$ has a due date d_i and a given absolute value of profit ω_i for its processing. The profit does not depend on the completion time of the task if it is not tardy in regard to its due date. Otherwise, the planning system's profit for this task is zero;

5) subject to the conditions: all products have due dates d_i. We need to minimize the total cost (penalty for the planning system) both for earliness and tardiness in regard to the due dates.

The Problem Solving Methodology. The problem in this formulation is a multi-stage scheduling problem (MSSP). Experts together with specialists in applied mathematics should present the initial technological process in the form of an MSSP adequate to the actual production process. Examples of such presentation of MSSP are given in [10, 11].

The optimization problem in this formulation cannot be solved efficiently. We cannot obtain exact solutions of MSSP because of its practical complexity. Approximate solutions basically converge to a step-by-step optimization which does not take into account the possibility of searching for a global optimum by a given criterion. Therefore, an hierarchical approach to the planning problem was proposed in [7–9] containing the following three levels.

Level 1: preliminary (predictive) planning. This level includes:

a) building a model of technological aggregation to reduce the problem's dimension. This is the aggregation of the original precedence graph to the level of multi-resources (stable groups of resources working together) and aggregated jobs construction (combining related operations of the same product executed on the same multi-resource). The processing time of an aggregated job is determined by its critical path in this multi-resource;

b) building a model of design aggregation. This is the graph of critical paths of the products processing with common vertices. In the graphs where each vertex has a processing time, the critical path is the route of the maximum total length. Procedures for finding a critical path in the graph are discussed in detail in [8]. A graph of critical paths constructed by such rules has a smaller size, since it includes only vertices on critical paths. Thus, we aggregate the model to a "single machine" representation. Some aggregated jobs may process on multi-resources that require a setup (preparatory work) to process jobs with different characteristics. We combine such jobs, according to certain rules, into common aggregated job if such jobs do not require the setup of the multi-resource when changing one job to another. We indicate this on the precedence graph by common vertices. In this case, the setup is required only at the beginning of the schedule and each time when the multi-resource switches from processing "common vertex" jobs to other aggregated jobs;

c) TWCTZ problem solving [7]. This problem serves to determine the priorities of the products and the processing order of the aggregated jobs. This, in turn, is the basic information for solving problems on the second and third levels of the model. It was shown in [7] that we can approximate any of the five basic criteria by a TWCTZ problem with corresponding weighting coefficients. The algorithm is described in [7], its modification in [10]. As a result, we obtain a priority-ordered sequence of aggregated jobs with a breakdown to maximum priority subsequences (MPSS).

Level 2: coordinated planning. It includes:

a) preliminary distribution of the aggregated jobs of the constructed graph of critical paths. We break down the common vertices to clarify the information about their combining at the distribution stage. To implement the coordinated planning, we have developed the following distribution algorithms [7, 10]:

- compact schedules construction (algorithm 1). We distribute the products from the beginning of the planning period and minimize their completion times by this algorithm;
- nondelay schedules construction (algorithm 2). We distribute the products from their due dates set by the customers;
- construction of schedules that ensure highest priority products processing in the specified due dates (algorithm 3);

b) redefining the set of common vertices according to the actual distribution information. If the set of common vertices has changed, then we rebuild the model and construct a new sequence of aggregated jobs (re-solve the TWCTZ problem). To do this, we again perform the procedures starting with the building the graph of the critical paths of products. To build the graph, we use the set of common vertices obtained during aggregated jobs distribution;

c) complementing the priority ordered sequence obtained as a solution of the TWCTZ problem with aggregated jobs which lie outside of products' critical paths. We assign corresponding MPSS numbers to the added jobs;

d) distribution of the obtained schedule among multi-resources with binding to the planned period. We do it using one of the above distribution algorithms, with such exceptions:

- we perform the algorithm on a initial precedence graph of the aggregated jobs;
- we partition the aggregated jobs into batches (the number of iterations is equal to the number of batches, we perform the iterations for batches instead of one iteration for aggregated jobs with full processing time);

e) as a result of multiple performing of actions described above, we generate a whole series of possible feasible plans that differ in a specific type of criterion, due dates, weight coefficients, manufacturing technology. Experts evaluate obtained plans (alternatives) in different contexts and choose the best plan to pass it to the third level of the model. The modified Analytic Hierarchy Process can be used to evaluate the plans [12]. Its use allows to make a reasonable choice of production plan from the set of feasible ones under conditions of uncertainty.

If we could not obtain a plan satisfying the specified requirements, the informational model of the first level is subject to adjustment. We can exclude or add new products, purchase new equipment, change the production technology, etc.

Thus, after performing the first two levels of the planning model, we obtain:
the optimal portfolio of orders as the experts exclude from the execution some products or their parts that violate the customer's due dates (they do the exclusion if purchasing an additional equipment or delay products is undesirable);
the coordinated plan of the aggregated jobs processing in multi-resources approved for implementation by the experts;
the due dates in the MSSP are determined by the completion times of the products in the approved plan.

Level 3: exact (operational) planning, includes [10]:

a) disaggregation of multi-resources and aggregated jobs to the level of the initial technological model;

b) the most compact operational plan is an arbitrary feasible (not violating the due dates we got at the second level of the model) solution of the MSSP by the criterion of maximizing the start time of the earliest job. An example of methodology to solve an MSSP is given in [10].

Thus, after performing the first two levels of the three-level model – independently of the production type considered, whatever is the original production technology, and no matter how the MSSP is implemented – we reduce the planning problem solving for any of the five above optimality criteria to obtaining a feasible MSSP solution for the criterion of maximizing the start time of the earliest job. For different optimality criteria, only the due dates in MSSP vary. The due dates are determined by the coordinated planning algorithm at the second level of the model. And they, in turn, depend on the priority-ordered sequence of aggregated jobs obtained as a solution of the TWCTZ problem.

Thus, the efficiency of the MSSP solution depends on the efficiency of the TWCTZ problem solution. Now we justify the efficiency of the three-level model for planning of arbitrary objects with a network representation of the technological process with optimization according to any of the above five criteria.

Efficiency research of TWCTZ problem solving algorithms. *The TWCTZ Problem Statement* [10]. A partial order on the set of tasks $J = (j_1, j_2, \ldots, j_n)$ is given by an oriented acyclic graph G. We know a processing time l_i for each task j of the graph G. Each terminal vertex (without successors) of the graph has a weight ω_j. Other vertices have zero weight. We need to find a sequence of tasks that minimizes the functional: $\sum \omega_i C_j \to \min$ where $\widetilde{N}_j$ is the completion time of a task j. Here, the tasks mean aggregated jobs. G is the graph constructed on the products' critical paths. The common vertices in the graph indicate common aggregated jobs for different products.

Exact algorithm for TWCT problem solving is given in [13], polynomial approximation algorithm for TWCTZ problem solving see [7].

To study the efficiency of TWCTZ problem solving algorithms, we have developed two generators of benchmark instances. The first one generated arbitrary precedence graphs with a given completeness $g = e/(n(n-1)/2) \times 100\%$ where e is the number of arcs in the graph and n is the number of vertices. We generated the graphs taking into account the required percentage k of the number of terminal (weighted) vertices. The second generator determined the weights of the terminal vertices and the processing times of all jobs. The parameters were set in such ranges:

- $n = [500, 750, 1000, 3000, 5000, 10000]$;
- $g = [2, 5, 7, 10, 15, 25, 50, 75, 90, 95]$;
- $k = [5, 10, 20, 30, 40, 50]$;
- $\omega_j \in [0, 10]$ (uniform distribution);
- $l_j \in [0, 100]$ (uniform distribution).

We generated 20 instances for each dimension and parameters g and k. All 7200 generated instances were solved both by the exact PSC-algorithm [13] and the approximation algorithm from [7] on a computer with 1 GHz processor frequency. Then we averaged the data for all values of the parameter k. The solving results are summarized in Tables 1–4.

Table 1 – The average time to solve the problem by exactPSC-algorithm (in seconds)

g \ n	500	750	1000	3000	5000	10000
2	0.732	2.677	6.723	226.110	1159.41	10654.5
5	0.518	1.896	4.760	160.087	820.863	7543.40
7	0.463	1.693	4.250	142.955	733.018	6736.13
10	0.387	1.417	3.558	119.684	613.692	5639.57
15	0.294	1.077	2.703	90.910	466.152	4283.74
25	0.201	0.734	1.844	62.020	318.018	2922.45
50	0.117	0.428	1.076	36.176	185.497	1704.64
75	0.076	0.277	0.694	23.353	119.746	1100.42
90	0.081	0.298	0.747	25.125	128.831	1183.91
95	0.030	0.110	0.277	9.305	47.712	438.45

Table 2 – The average time to solve the problem by the approximation algorithm (in seconds)

g \ n	500	750	1000	3000	5000	10000
2	0.075	0.273	0.686	23.086	118.38	1087.83
5	0.055	0.200	0.501	16.850	86.401	793.991
7	0.050	0.182	0.456	15.348	78.699	723.208
10	0.043	0.157	0.393	13.228	67.829	623.316
15	0.034	0.125	0.313	10.528	53.983	496.085
25	0.025	0.093	0.233	7.840	40.198	369.405
50	0.018	0.066	0.165	5.538	28.394	260.933
75	0.014	0.050	0.125	4.203	21.550	198.037
90	0.016	0.058	0.147	4.929	25.276	232.278
95	0.006	0.022	0.056	1.876	9.620	88.401

Table 3 – The percentage of approximate solutions that coincide with the exact solution

g \ n	500	750	1000	3000	5000	10000
2	100.0	100.0	100.0	100.0	100.0	100.0
5	100.0	100.0	100.0	100.0	100.0	99.2
7	100.0	100.0	100.0	99.2	99.2	99.2
10	100.0	99.2	100.0	100.0	99.2	99.2
15	99.2	99.2	98.3	99.2	98.3	98.3
25	99.2	98.3	99.2	98.3	98.3	98.3
50	98.3	97.5	98.3	98.3	97.5	97.5
75	98.3	98.3	97.5	97.5	97.5	96.7
90	97.5	96.7	96.7	97.5	96.7	96.7
95	96.7	95.8	95.8	95.8	95.8	95.0

Table 4 – The average percentage of deviation of approximate solutions from the optimum

g \ n	500	750	1000	3000	5000	10000
2	–	–	–	–	–	–
5	–	–	–	–	–	0.56
7	–	–	–	0.79	0.80	0.83
10	–	0.59	–	–	1.04	0.92
15	0.71	0.81	1.68	0.80	1.65	1.25
25	0.78	1.60	0.93	1.59	1.52	1.63
50	1.30	2.41	1.83	1.71	2.49	2.57
75	1.57	1.70	2.45	2.58	2.53	3.45
90	2.33	3.19	3.35	2.51	3.32	3.48
95	3.10	3.87	3.97	3.70	4.35	4.94

On average for all runs, the solutions obtained by the approximation algorithm coincided with the solutions obtained by the exact PSC-algorithm for TWCT problem solving [13] in 98.47 % of cases.

To illustrate the dependence on the parameter k, we show in Table 5 the average solving time by the PSC-algorithm for 500 jobs instances.

Table 5 – Dependence of the solving time by the PSC-algorithm (ms) on the parameter k at $n = 500$

g \ k	5	10	20	30	40	50
2	304.216	350.791	563.722	791.327	1140.63	1238.54
5	57.960	132.291	313.470	583.533	929.815	1090.51
7	44.850	102.424	273.629	486.206	917.665	950.243
10	34.107	87.600	219.364	405.449	818.008	758.752
15	28.020	68.996	175.420	299.893	546.218	646.184
25	21.349	60.836	141.692	224.383	344.195	411.478
50	17.429	40.751	91.327	140.863	171.903	239.971
75	15.904	30.889	66.413	96.053	124.591	119.478
90	13.203	27.871	55.428	228.646	–	–
95	12.422	24.632	53.258	–	–	–

Thus, we can conclude that:

1. The approximation algorithm allows to solve real practical problems of large dimensions.
2. In comparison with the PSC-algorithm for the general case of the TWT problem, the solution time of the approximation algorithm is an order of magnitude shorter.
3. With an increase in the graph's completeness, the solving time decreases, but the accuracy of the solution of the approximation algorithm decreases.
4. The average percentage of the deviation of the solution obtained by the approximation algorithm from the optimum is 1.49 %.
5. The conditions of the polynomial component of the PSC-algorithm are not met on average for 1.53 % of the total number of instances;
6. With an increase in the percentage of the number of terminal vertices, the time to solve increases according to a law that is close to linear.

An example to the TWCT problem solving. Consider the graph of the critical paths of the three products shown in Fig. 1. We give the initial feasible sequence of tasks in Table 6. In tables 6 and 7, N is the vertex number in the graph of critical paths of products.

The exact PSC-algorithm for the problem solving is based on permutations of the following structures: a chain, an elementary construction, constructions K_1 and K_2 [13]. We build the constructions in the process of a problem solving on the basis of weighted tasks. We move these structures into earlier positions in the current sequence in accordance with their priorities. The interval of their move, as well as the combinatorics of their construction during the problem solving, is determined by common vertices which relate in the precedence graph with the structures under consideration. Obviously, the smaller the number of common vertices, the less complexity of the algorithm execution. The above mentioned structures for permutations are not formed on zero-weighted tasks. Thus, the complexity of the problem solving is determined by the number of vertices loaded with weight and the number of common vertices.

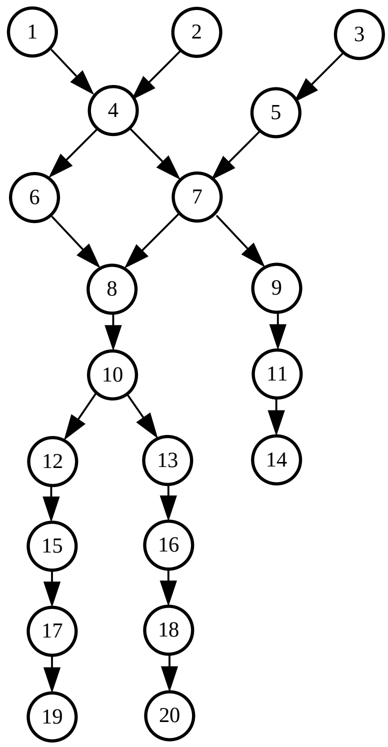

Fig. 1. Graph of products' critical paths

Table 6 – Initial feasible sequence for TWCTZ problem solving

N	ω_i	l_i	Common?	C_i
1	0	12	No	12
2	0	14	No	26
4	0	15	Yes	41
3	0	11	No	52
5	0	14	No	66
7	0	32	Yes	98
6	0	22	No	120
8	0	25	Yes	145
10	0	19	Yes	164
13	0	7	No	171
16	0	21	No	192
18	0	18	No	210
20	30	18	No	228
9	0	28	No	256
11	0	16	No	272
14	20	8	No	280
12	0	6	No	286
15	0	19	No	305
17	0	16	No	321
19	10	17	No	338

The priority-ordered sequence of aggregated jobs with a breakdown to MPSSes is the result of the problem solving by an exact algorithm [13]. We show it in Table 7.

The result of the problem solving by the approximation algorithm [7] coincides with that obtained by exact algorithm.

Table 7 – Initial feasible sequence for TWCTZ problem solving

N	ω_i	l_i	Common?	C_i	f_i
1	0	12	No	12	
2	0	14	No	26	
4	0	15	Yes	41	
3	0	11	No	52	
5	0	14	No	66	
7	0	32	Yes	98	
9	0	28	No	126	
11	0	16	No	142	
14	20	8	No	150	3000
6	0	22	No	172	
8	0	25	Yes	197	
10	0	19	Yes	216	
13	0	7	No	223	
16	0	21	No	244	
18	0	18	No	262	
20	30	18	No	280	8400
12	0	6	No	286	
15	0	19	No	305	
17	0	16	No	321	
19	10	17	No	338	3380

The optimal functional value is 14780.

As a result of the PSC-algorithm execution, the procedures associated with the enumeration of various constructions were not performed.

The approximation algorithm [7] is based on the algorithm for a series-parallel graph [7]. In contrast to the exact algorithm, the enumeration of various cases of the structures construction is excluded in advance in it. The solutions obtained by both algorithms coincided since in the solving process the conditions for the structures' formation and their enumeration were not fulfilled.

Conclusions. We have shown that whatever was set the initial manufacturing technology for products, an adequate scheduling model should be constructed to obtain the production operational plan. And then, obtaining a good operational plan for any of the five basic criteria of optimization reduces to a single uniform problem. We have to build a feasible schedule by the criterion of maximizing the start time of the earliest job for the multi-stage schedule problem adequate to the initial technological process of the production or object under consideration. For various optimization criteria, we have to change only the due dates determined at the second level of the three-level model as the completion times of the products during the coordinated planning. Since the efficient due dates depend on the efficient solution of TWCTZ problem, the three-level planning model is efficient when its first level is efficient.

We have shown that the approximation algorithm for TWCTZ problem solving [7] allows to solve real practical large size problems (we checked dimensions of up to 10000 jobs). The solutions obtained by the approximation algorithm coincided with those obtained by the exact PSC-algorithm for TWCT problem solving [13] in 99.97 % cases. Hence, the polynomial algorithm proposed in [7] for TWCTZ problem, due to the presence of weights only on the terminal vertices of the job precedence graph, statistically significantly yields exact solution. We propose to use it in planning for arbitrary objects with a network representation of technological processes.

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